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GeoGebra: A Mediating Artifact to Minimize Students' Misconceptions in Learning Function Concepts

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Abstract

The quadratic function concept is the most significant but problematic to teach. Thus, it needs further consideration in order to provide the maximum benefit to students. The recently revised EGECF believes in integrating technology into teaching and learning to cope with 21^{st}-century advancements. GeoGebra is the most important mathematical software that joins two pillars of mathematics representation i.e. geometry and algebra. This study investigated the effect of GeoGebra software on overcoming students' misconceptions and attitudinal change in learning the quadratic function concepts. The study used a quasi-experimental, non-equivalent control group design, and sequential explanatory method to compare performance between grade nine students in two groups. The population for this study consisted of 473 9^{th}-grade students in nine classrooms in two secondary schools of Worabe Town Administration (WTA), Ethiopia. The schools were repeatedly scored low in national examinations. Two samples of sizes n = 42 (experimental) and n = 45 (control) groups which are intact classrooms drawn randomly from classrooms in the two schools. The study was guided by activity theory and conducted in 2022/23. Pre-test/post-test, questionnaire, and interview were used to collect data. The findings of the study revealed that pre-test scores for both groups were comparable at the outset by determining the baseline knowledge since T (85) = 0.0135 and the p-value 0.81077 is greater than 0.05. The result of the post-test indicated that GeoGebra software has an important role in reducing students' misconceptions of concepts in quadratic functions. Besides, students had a positive attitude towards the GeoGebra applet. Moreover, the study showed that the application reduces teachers' and students' efforts on routine and procedural tasks to strategic and conceptual aspects. Hence, the study recommended that mathematics teachers be introduced to the software and experience its effects on themselves and their students. Furthermore, concerned bodies must take the initiative to incorporate GeoGebra software into the curriculum, teacher education programs, and in-service courses for mathematics teachers. This study has potential limitations and suggests comprehensive studies using other mathematics topics, far larger randomized sample sizes, at different schools of different composition and socio-economic status, which reflect the entire zone, region, or country level.

1 Introduction

One of the crucial challenges in Ethiopia is the poor performance of students in general and mathematics in particular (Sebsibe *et al*., 2023) for various reasons such as misconception during instruction delivery, the negative attitude of students towards mathematics, ineffective teaching strategies, and less practical aspects of the lessons (Walelign, 2014). Information about students' prior knowledge and the cognitive features that come along

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with it can assist us in knowing students more easily and provide an opportunity to design appropriate interventions. The literature revealed different definitions of the term "misconception". Identifying and minimizing misconceptions made by students about mathematical concepts is important. Students' misconceptions and attitudes are basic concerns of this study.

According to Murray *et al*. (1990), misconception can be observed at every age and educational level. It is a perception or conception that is incompatible with the opinion commonly agreed on by experts on a particular subject. For Michael (2002), a misconception occurs when there is an inconsistency between the concept that we want students to construct and the mental model that they build in their minds. According to Smith *et al*. (1994), a misconception is a student's conception that results from a systematic pattern of errors observed in their performance. In other words, a misconception is information that contradicts currently accepted scientific theories (Clement, 1993).

Research shows that interactive learning plays a prominent role in students' understanding of concepts. For instance, Aslam and Kingdon (2011) posits that teachers' classroom practices and the teaching process, such as students' participation, matter a lot in students' learning. The literatures, (for instance, Elia *et al*., 2007; Federal Democratic Republic of Ethiopia Ministry of Education, 2020; Melissa *et al*., 2023; Misini & Kabashi, 2021; Övez, 2018) show that the integration of technological tools like GeoGebra has the potential to improve the effectiveness of the teaching and learning of mathematics in general and the quadratic function concept in particular (Melissa *et al*., 2023; Misini & Kabashi, 2021; Övez, 2018). Though the recently revised Ethiopia General Education Curriculum Framework (EGECF) believes in integrating technology into teaching and learning to cope with 21st-century advancements (Federal Democratic Republic of Ethiopia Ministry of Education, 2020), its implementation has been slow (Sebsibe *et al*., 2023). In this respect, this study intends to contribute towards narrowing the perceived gap in the existing literature by providing some insights into learning using GeoGebra to improve misconceptions in mathematics in general and quadratic function in particular in the context of secondary schools in Ethiopia.

One of the fundamental and core ideas in mathematics is the idea of a function. It emerges from the urge of humans to uncover patterns among quantities, which is as ancient as mathematics (Elia *et al*., 2007; Gagatsis & Shiakalli, 2004). Any student who wants to better comprehend calculus and pave the way for progression in scientists, engineers, and mathematicians must have a firm grasp of the function notion (Carlson & Oehrtman, 2005). Likewise, understanding the graph of the quadratic function is very important for students because it is a fundamental subject before students work with functions with higher degrees and more complex polynomial functions (Septian *et al*., 2020; Suzanne *et al*., 2015). Summarizing students' difficulties in learning concepts in function Övez (2018, p.3) mentioned that "solving and interpreting the graphic due to accepting the function's graphic as a fixed object" is among the misconceptions that students form in function. Besides, the quadratic functions concept is the most problematic to teach (Övez, 2018). Therefore, the concept needs to be discussed further so that students' best benefit from it.

According to Gebremeskel *et al*. (2018), negative attitudes toward mathematics and misconceptions in instruction is one of the reasons, among many different factors, for the poor performance of students in mathematics nowadays in Ethiopia. It is widely recognized that the traditional approach to teaching mathematics is not effective in reducing difficulties and misconceptions. Besides, in every discipline, concepts are the basis for further learning and development of a subject. This is especially true with mathematics, which is by its very nature highly sequential. If prerequisite concepts are not established clearly, understanding the following concepts is unlikely. As mentioned above, the function concept is a base for discussing and applying many advanced concepts in mathematics. Thus, designing a preventive strategy to overcome observed difficulties and improve students' attitudes toward mathematics is important.

This study is part of subsequent studies aimed to

design an instructional approach that is supposed to enhance students' understanding of concepts in mathematics. At this stage, it is intended to examine the effect of GeoGebra-supported instruction compared to the traditional instruction on students' misconceptions and attitudes toward learning the quadratic function concept.

In line with this, the study addressed the following three specific research questions:

1. to what extent does Geogebra-supported instruction enhance students' capability to sketch Quadratic functions graphs?
2. to what extent does GeoGebra-supported instruction enhance students' abilities to identify the influence of coefficients and the constant on the shape of quadratic function graphs?
3. does applying GeoGebra software in teaching and learning quadratic functions develop a positive attitude in students?

Improving mathematics results in secondary schools in Ethiopia is a contemporary problem to which practical solutions are yet to be found. This study has sought to contribute in this regard by exploring alternative teaching and learning methods, especially for topics traditionally regarded as problematic to both teachers and students, such as functions and their graphs.

Theoretical Framework

Central to this study is how learners use artifacts, particularly virtual manipulative in the form of interactive, dynamic GeoGebra applets, to enhance their understanding of certain mathematical concepts i.e. in the study of quadratic functions. The Activity Theory underpins this study because computers potentially impact or mediate learning.

The Activity Theory: The activity theory states that an activity consists of a subject and an object, which are connected through a tool. The subject is a learner or learner engaged in the activity, and the subject holds the object and serves to motivate and direct the activity (Albusaidi, 2019). Culture, thought patterns, and language are only a few examples of the various material and mental instruments that can be used in mediation (Albusaidi, 2019). Figure 1 displays the interplay between a subject (human agent) and an object as mediated by tools or signs called Vygotsk's triangular model of a complex, mediated act (Mudaly & Uddin, 2016).

Using the activity theory, analysis, according to Leont and Laureate (1978, p. 37), considers three levels:

- assessing the activity and determining its purpose,
- analyzing the action and its aim, and
- studying the operation and its circumstances.

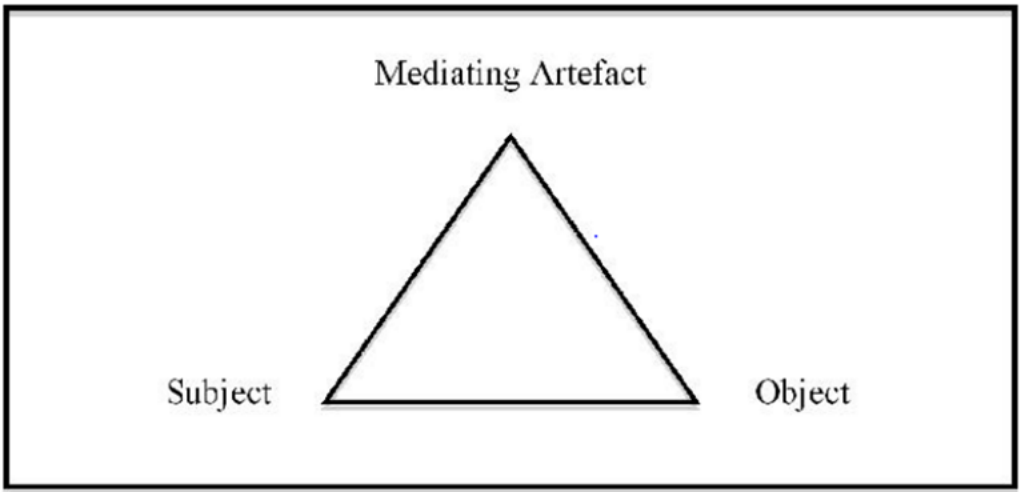

Figure 1: The interplay between a subject and an object as mediated by tools or signs

These are then combined into the actions the students will carry out to achieve a goal or perform better in courses.

During learning using tools like GeoGebra, the activity involves operating procedures on the computer screen including dragging sliders to manipulate the applets to visualize the outcome of a certain variable slide. Dragging the sliders achieves the purpose of comprehending how the function transforms as a result of changes to the function variable(s), resulting in a visual interaction with previously abstract notions. Conditions refer to the computer-driven environment in which this activity is conducted and how students use these applets to learn, explore, confirm, challenge, prove, and draw conclusions (Albusaidi, 2019).

The usual classroom system is described as the student, teacher, and subject matter interaction triangular model. Here the role of the teacher is a mediator of students' understanding of the subject matter. For teaching involving interventions like the present one, Vygotsky's Zone of Proximal Development (ZPD) enhances learning in collaboration with the teacher as facilitator, and fellow students as peers. The integration of GeoGebra as a scaffolding tool during teaching and learning does not substitute the teachers. Still, teachers must be aware of their students' cognitive demands and provide assistance and support in following those needs. The support can be delivered by the classroom instructor or a more capable individual (Guseva & Solomonovich, 2017; Siyepu, 2013). This collaborative social interaction benefits both higher and lower-ability students. To do this, they must complete certain tasks without entirely depending on their teachers but aim to attain a certain objective. According to Tinungki (2019, p. 134), students use their prior knowledge to carry out the task without guidance. Hence, the application of the activity theory in a GeoGebra integrated instruction (Mudaly & Uddin, 2016, p. 199) suggests rules, community, and division of labour in addition to the subject, object, and tool as a structure of an activity system.

While the following are explanations for each component of the system figure 2 displays the structure of the activity theory which is the interaction between components of GeoGebra integrated instruction.

- the subject stands for the individual(s) whose perspective is taken in the analysis of the activity. To do so, their prior knowledge of functions (definition, basic operations, and properties i.e. the change in behavior concerning change in some parameters) and computer manipulation skills are required.
- the object (or the objective) is the intended goal of the activity within the system. Visualising the effects of a variable change to the functions in different forms and drawing generalizations to a function given a standard form is expected.
- tools are internal or external mediating artifacts that help to achieve the outcomes of the activity. Engagement with GeoGebra applets to enhance understanding of the concepts will be performed.
- the community comprises one or more people who share the objective with the subject. Learners with different abilities and interests and the teachers are the main communities of this practice.
- rules refer to house rules, norms, and agreements implicitly or explicitly agreed upon that constrain actions and interactions within the activity system. The evaluation criteria, expectations of the teacher, the teacher house rule, rules of the school, and discipline of the computer lab are included.
- division of labor discusses how tasks are divided horizontally between classroom community members and refers to any vertical division of power and status. The roles and responsibilities of students (especially in their group work) and teachers, cooperation among teachers, and the support of the lab assistant are all included (Mudaly & Uddin, 2016).
- Outcome: conceptualization of transformation of functions. The reflection of the use of virtual manipulation in the teaching-learning process for the learning of students and instruction.

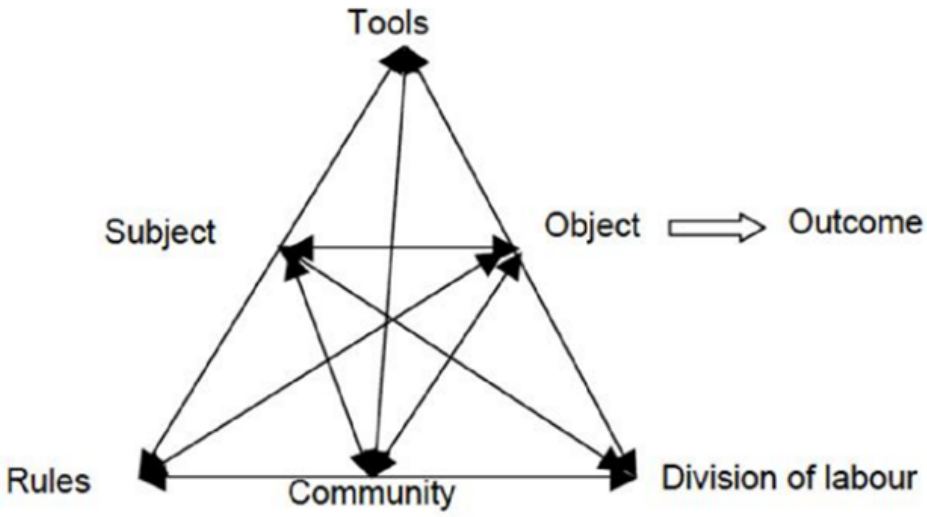

Figure 2: The Structure of the activity theory

Overview of the GeoGebra software

GeoGebra was designed by Markus Hohenwarter (Hohenwarter & Jones, 2007; Zengin *et al*., 2012) as open-source dynamic mathematics software that incorporates geometry, algebra, and calculus into a single, open-source, and user-friendly package. GeoGebra's interface provides two presentations of each mathematical object, one in its graphic (geometry) window and the other in its algebra window, hence the name GeoGebra. GeoGebra. It is accessible in multiple forms: its desktop applications for Windows, Mac OS, and Linux, its tablet applications for Android (smartphone), iPad, and Windows, and its can be used either online or offline (Majerek, 2014).

GeoGebra has many possibilities to help students experience intuitive feelings and visualize adequate mathematical processes. Students can connect symbolic and visual representations using the tools provided by this software, allowing them to explore a wider range of function types (Diković, 2009). A change of an object in one of these windows will immediately result in a change in the other window thus increasing the learner's ability to recognize significant cognitive relationships. It is recognized as a tool that raises students' performance and critical thinking as it involves experimental and guided discovery learning (Övez, 2018). Besides, students grow positive attitudes toward learning as it makes the teaching and learning process more conducive (Dockendorff & Solar, 2018; Övez, 2018).

The software provides a geometry window or working area, a toolbar, an algebra window, an input field, a menu bar, and a navigation bar. Although GeoGebra best provides a platform for the teaching of geometry, it offers equally well features for the teaching of algebra particularly in functions and graphs. Functions can be defined algebraically and then changed dynamically. For instance, by entering the equation, the resultant graph is immediately produced in the geometry area, and the corresponding algebraic component of the graph appears simultaneously in the algebra window. In this study, we use the GeoGebra applet to work with activities in quadratic functions. Figure 3 is a screenshot showing the GeoGebra window that connects algebra and geometry. As shown in the figure, simply by typing a single quadratic function of the form $f(x) = ax^2 + bx + c$ or $y = ax^2 + bx + c$ in the input area, one can see both algebraic, to the left side, and the graph, to the right side.

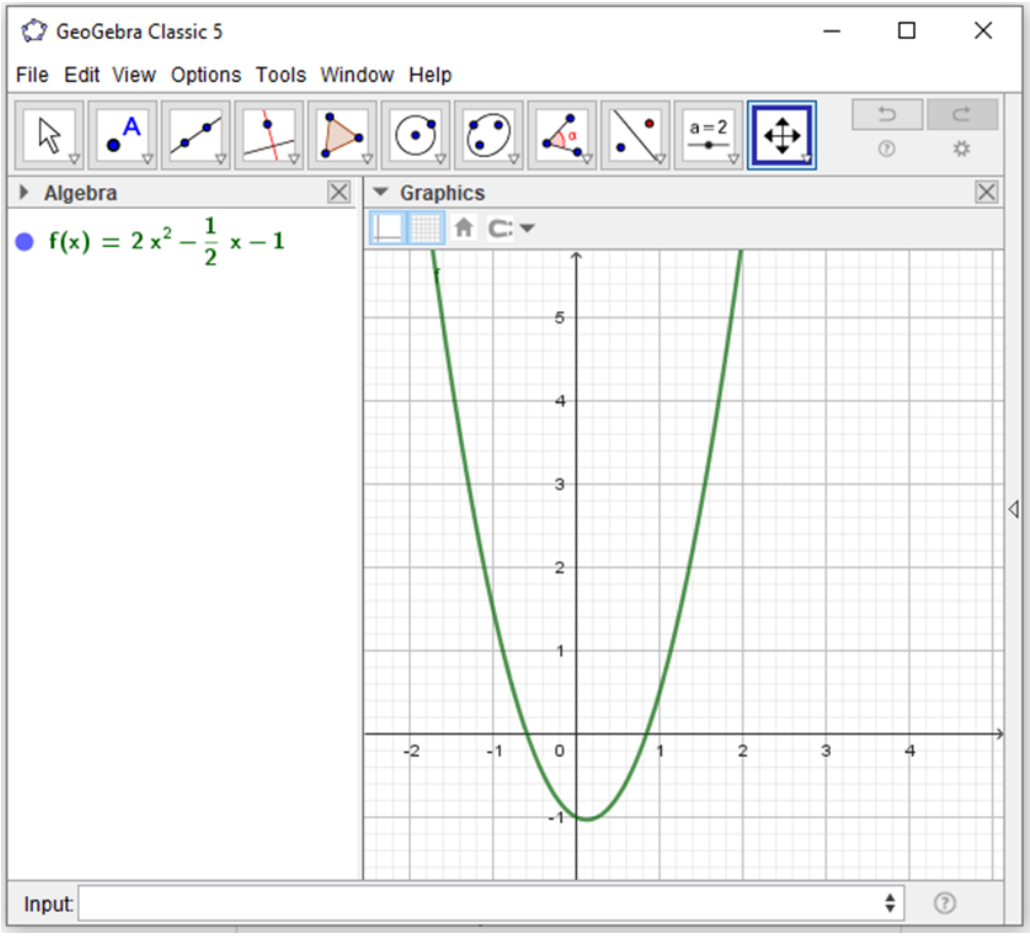

Figure 3: Screenshot from a GeoGebra window

The benefits of GeoGebra, include its ability to quickly and accurately illustrate geometry, to represent multiple functions with different colures or font sizes, and to provide animation and movement features that assist in visualizing the effect of change in parameters on the behavior of the function. For instance, what is the effect of changing the sign of a in $y = ax^2 + bx + c$? This study focused on the effect of using this software on students' ability to overcome misconceptions and enhancing attitude toward learning.

2 Method and Materials

The sequential explanatory and quasi-experimental methods of non-equivalent comparison group design were employed to address research questions. The study was conducted in Worabe Town Administration (WTA) in the Silte Zone, part of a regional state in Ethiopia. Among four schools in the Town administration, two schools that repeatedly scored low in national examinations as compared to other schools in the Zone were used. Hence, the population for this study was all Grade 9 students in the two schools which consisted of 473 (210 male and 263 female) grade 9 students. Poor performance is usually due to making errors during examination and errors are mostly due to misconceptions in the subject matter which is why the researcher focuses on purposely poor performance as criteria to select samples. A sample of two intact classroom 9th grade students one from each school were randomly selected with a total of 87 students, which constitute around 18.4% of the population. One group (N=42) was assigned to be the experimental group and the other (N=45) was assigned to be the control group. The study was conducted in the 2022/2023 academic calendar.

This study mainly used quantitative data through pre/post-tests to compare the performance of the two groups and a questionnaire for attitudinal at-

tributes for the experimental group. Interviewed was also used to supplement and triangulate quantitative data. The pre-test was based on basic concepts of function in general. It was assumed that all learners were used their experience of mathematics content on a function to answer the pre-test. To compensate for the non-random assignment of students to the control and experimental classes, the pre-test was used to determine if the classes were comparable at the outset by determining the baseline knowledge or preparedness for learning the topic of quadratic function. It comprised of 18 multiple-choice questions covering basic function concepts from the grade 9 mathematics syllabus. The post-test comprised a total of 16 items of mixed type (all of which had their scoring key written at the end of the instruction) items which involved multiple choices, matching, essay, and procedural items developed by the researchers from National examination booklets of previous years, grade 9 mathematics syllabuses, textbook and minimum learning competency, the literature, and model examinations of one of the special board secondary school at WTA.

The question papers were given to five experienced mathematics teachers and two curriculum experts to criticize and comment on the items. Besides, a pilot study (N = 40), was carried out at a remaining school to check the reliability and validity of the research instruments, and statistical viability. Based on the expert validation and the pilot test result the initially designed 25-item pre-test was reduced to 18 items after detecting the deficits that need modification and rejecting items that have unacceptable levels of difficulty (item difficulty index 95% and 5% respectively). The same procedure was applied for the post-test pilot but no item was rejected except modifications in three items' distractors which had poor discrimination power. Table 1 is a summary of the post-test items in themes based on the objective of the topics in the textbook.

Table 1: Post-test items into themes

Theme	Expected Outcome	Items addressing the theme
1	Interpret and use the vertex form of a quadratic function	1, 3, 4, 6, 8, 12, 13, & 14
2	Interpret and use the standard form of a quadratic function	2, 7, 9.1, 9.2, 15, 16.2.1, 16.2.2, 16.2.3, & 16.2.4
3	Interpret and use the graph of a quadratic function	3, 4, 7, 10.1, 10.2, 10.3, 10.4, 11.1, 11.2, 11.3, & 11.4
4	Applying the shifting rules to graph quadratic functions	5, 8, 16.1.1, 16.1.2, 16.1.3, 16.2.1, & 16.2.2

The questionnaire was used to analyze student's perception and motivation towards GeoGebra software in teaching mathematics. It is perception attributes based on five-point Likert scales (Strongly agree, Agree, Undecided, Disagree, and strongly disagree). The questionnaire items designed by the researchers focused on only motivation and perception attributes relevant to teaching and learning process i.e. participation during lesson delivery, concentration during lesson delivery, enjoyment of class activities, self-confidence, content mastery, and recommendation or preference of the teaching/learning method (adapted from Praveen & Leong (2013)). The interview items were designed after carefully scrutinized and discussed between the researchers. The interviews were conducted on a face-to-face basis and the responses of the interviewee were recorded and taken as a note.

The experimental group was instructed using GeoGebra-supported instruction in the school's computer laboratory, and the control group was instructed using the traditional method. Before the actual administration of research instruments and data collection, researchers visited the sampled schools to check the ICT infrastructure and suitability for the research and verbally communicated and explained the purpose of the study and minimized the Hawthorne effect. After consensus and classes had been arranged well for the research purpose, the pre-test was administered for both groups. Then the delivery of instruction continued with the aid of an overhead projector and after having had two days of introduction about the Geogebra software utilization for the experimental group. Two weeks of instruction (10 days) was given by the second researcher for both groups. While both instructions

are based on the content of the textbook, the difference is the method of working activities. In the experiment, the teacher uses a projector to guide on how to perform the tasks using the GeoGebra, while for the control group uses the conventional approach.

After two weeks of instruction, the post-test was administered to both groups. The questionnaire was also administered to the experimental group following the post-test. After carefully scrutinizing the post-test, six participants from the experimental group who scored high, medium, and low were interviewed to supplement and triangulate quantitative data.

The reliability of the pre-test and post-test was established using data from the pilot study. Two reliability tests were computed, the Kuder-Richardson 20 (KR20) for the pre-test and Cronbach's Spearman-Brown formula for the post-test. Accordingly, the reliability value was as found 0.83 and 0.74 for the pre-test and post-test respectively which was acceptable (Gay *et al*., 2011). The reliability and validity of the questionnaire were verified by experts' comments including readability, feasibility, layout, style, and clarity of wording.

The data was jointly analyzed using descriptive statistics (mean, percentage frequency counts) and inferential statistics (independent t-test) methods. It was checked whether there was a significant effect on the students' misconceptions in the two groups. The statistical package for social sciences (SPSS 20) was use for the inferential analysis of the data.

In the study, the researchers attempted to fulfill all acceptable requirements about safeguarding and protecting the rights of all concerned. The researcher did this by requesting permission from the Zone Education Department and respective school principals through letters. The students were also fully aware of their involvement in the study and informed of their rights as participants. The participants' response was codded and their identity was not revealed in the study report.

3 Result

3.1 Baseline equivalency check

After checking the normality of the distribution in the pre-test, an independent t-test was computed and showed that there were no statistically significant differences between the experimental group and the control group in studying linear functions because the p-value 0.811 is greater than 0.05 indicating that the two groups were of comparable/similar ability in prerequisite knowledge before treatment; as such, any differences in studying quadratic function after treatment could be attributed to the treatment. Figure 4 shows the normality curve of the pre-test result for the two groups.

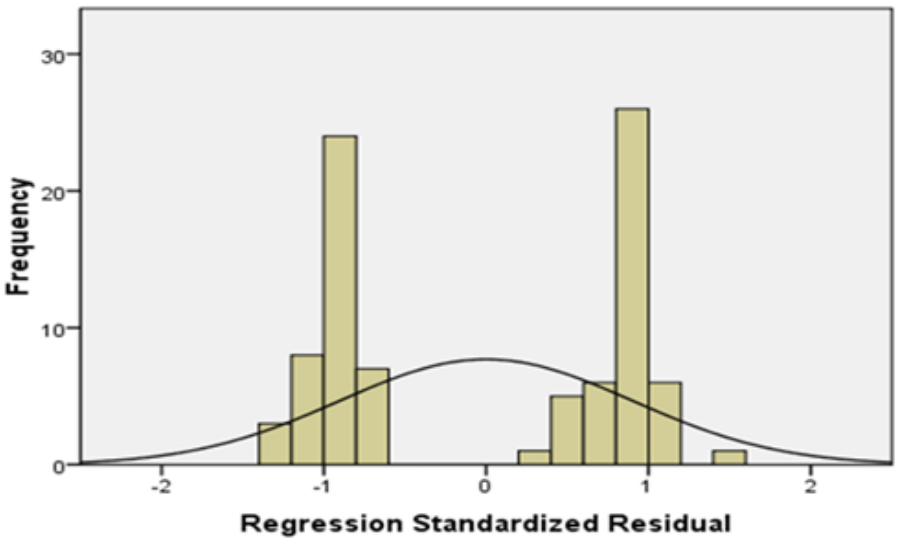

Figure 4: Normality curve of pre-test

3.2 Post-test results

Students Misconceptions

Scores in theme 1: Interpret and use the vertex form of a quadratic function

Table 2 shows cross-tabulation of students score for items in the first theme. There were eight items in this theme which are item number 1, 3, 4, 6, 8, 12, 13 and 14.

Table 2: Mean score of students for items in theme one

Theme one item numbers	Respondents Type							
	Control Group (N = 45)				Experimental Group (N = 42)			
	Missed the item		Answered the item correctly		Missed the item		Answered the item correctly	
	Count	%	Count	%	Count	%	Count	%
1	14	31.11	31	68.89	4	9.53	38	90.47
3	20	44.44	25	55.56	5	11.90	37	88.1
4	22	48.89	23	51.11	2	4.76	42	95.24
6	15	33.33	30	66.67	1	7.14	42	92.86
8	23	55.56	22	44.44	5	11.90	37	88.1
12	14	31.11	31	68.89	12	28.57	30	71.43
13	15	33.33	30	66.67	9	21.43	33	78.57
14	21	46.67	22	53.33	14	33.33	31	66.67
Aggregated mean		40.555		59.445		16.07		83.93

As portrayed in Table 2, for the items in this theme, the mean of correct respondents in the control group is 59.44%, and in the experimental group, it is 83.93%. The researchers examined in detail why this had happened while scoring the papers. Figure 5 shows scanned images of some of the students' work.

In Item 1 students were asked to find the vertex of the parabola $f(x) = 9(x+3)^2 - 10$, and some students responded as $(-3, 10)$ and $(3, 10)$, thinking that if the value of x in the bracket (-3) changed, the sign so does the value outside bracket (10). Similarly in question number three, students were asked about the minimum/maximum value of the graph of the function $y = 5(x-3)^2 - 2$. Some students think if the value before the bracket in the vertex form equation of quadratic function is positive, then it would have maximum and if negative, then minimum.

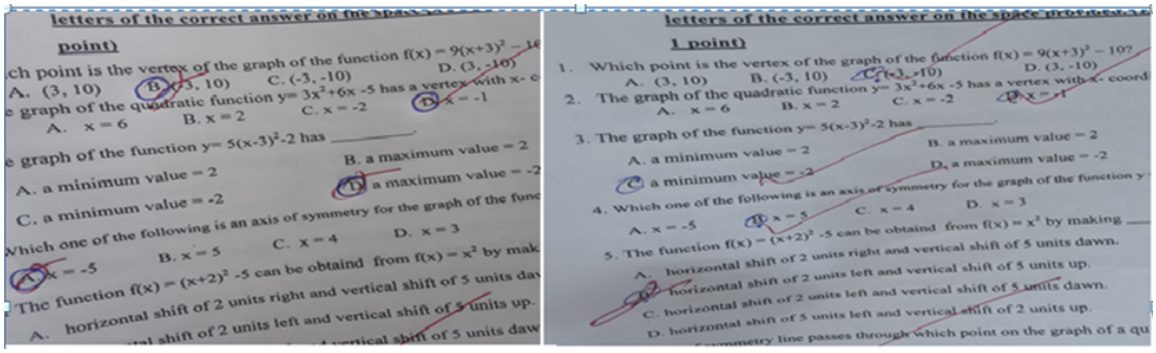

Figure 5: Scanned image of the students' works for items in theme one

To triangulate this issue, after arranging the scores in descending order, some of the respondents' six students (three from each group) were interviewed. Interviewees responded, "***Have you missed or got it right items in theme one? Why or why not?***".

Interviewees CG22, CG38, CG44, EG19, EG27 and EG33 (Where Letters CG and EG followed by

the numbers are codes for respondents from the control and the experimental group respectively) replied as follows:

CG22: "*Teacher, I missed most of the items related to vertex. I made my mistakes immediately while we discussed them with classmates after completing the exam. Wrongly I perceived as if it would have been taken opposite of the values in vertex form of equation of quadratic function to find vertex, vertical and horizontal shifts*". March, 27/2023.

CG38: "*I missed the items you asked, I understood well after the exam referring to my notes and worksheet solutions. In the equation* $a(x+b)^2+c$ *which is a vertex form equation, I have taken the signs directly without changing the sign of 'b' as* $(-b, c)$ *to find the vertex due to this I also missed the direction shifting. In question three* $(5(x-3)^2-2)$ *since 5 is positive I assume it has maximum and if negative it has minimum*". March, 27/2023.

EG19: "*I missed some of the questions because I did not attend 2 classes due to a social problem. But I tried to understand the missed portion from one of my classmates I don't know how I missed*". March, 28/2023

CG44: "*Oh, teacher; I missed items especially concerning vertex, vertical and horizontal shifts, and maximum and minimum values of a parabola in such a way that if b and c in the vertex formula are positive then the parabola shifts horizontally to positive x-direction and vertically to positive y-direction respectively and vice versa and the vertex taking directly as* (b, c) *considering the sign of 'b' and 'c' as it is*". March, 27/2023

EG27: "*I answered all the questions except item 14 because GeoGebra software helped me to visualize how the parabola shifts left or right and up or down by moving the slider so I was easily able to connect to the vertex formula. But failed to explain the axis of symmetry correctly*". March, 28/2023

EG33: "*Yes, I have answered all the questions, and no trouble was encountered in tackling the questions because the software was very helpful in understanding concepts in graphing quadratic functions. All the exercises and the worksheet questions helped me to exercise the Geeogebra well so that my exam result was also very good*". March, 28/2023.

From the above interview data in the control group, one can see students' misconceptions occurred due to overgeneralizations. As a result, the aggregated percentage of respondents in the control group for items in theme one is 59.44% whereas for the experimental group is 83.93 % indicating the intervention fevers the experimental group to reduce students' misconceptions. In general, in this part of the questionnaire items, the following misconceptions were observed: overgeneralization, exchanging the sign of the turning points, and failing to identify the terms "at what value of x have minimum/maximum" and "what is the minimum/maximum value".

Concerning this and as misconception had an inverse relation to the achievement of students the mean score and test of significance as measured in the T-test of items in theme one prevailed that among eight items five items namely 1, 3, 4, 6, and 8 are statistical ($p = 0.008 < 0.05$). Whereas the remaining items 12, 13, and 14 were not statistical since $p = 0.65 > 0.05$ indicating that the two groups have similar understanding in writing vertex form equations of quadratic function, identifying the parameter responsible for shifting the graph of quadratic function vertically up or dawn and defining the axis of symmetry of a parabola.

Students Score in Theme 2: Interpret and Use the Standard Form of a Quadratic Function

Table 3 shows the cross-tabulation of students' scores of items in the second theme. There were nine items to gauge this theme (items number 2, 7, 9.1, 9.2, 15, 16.2.1, 16.2.2, 16.2.3 & 16.2.4.).

Table 3: Mean score of students for items in theme two

Theme two item numbers	Respondents Type							
	Control Group(N = 45)				Experimental Group(N = 42)			
	Missed the item		Answered the item correctly		Missed the item		Answered the item correctly	
	Count	%	Count	%	Count	%	Count	%
2	15	33.33	30	66.67	5	11.90	37	88.10
7	22	44.89	23	51.11	10	23.81	32	76.19
9.1	19	42.70	26	53.30	9	21.43	33	78.57
9.2	19	42.70	26	53.30	7	16.67	35	83.33
15	9	20	36	80	1	2.38	41	97.62
16.2.1	9	20	36	80	5	11.90	37	88.10
16.2.2	11	24.44	34	75.56	1	2.38	41	97.62
16.2.3	8	17.78	37	82.22	2	4.76	40	95.24
16.2.4	14	31.11	31	68.89	1	2.38	41	97.62
Aggregate mean		30.77		69.23		10.84		89.16

As portrayed in Table 3, the overall mean percentage of 69.23% from the control group and 89.16% from the experimental group answered theme two items correctly and the experimental group surpassed the control group. But some students from the control group (33.33%, 44.89%, 42.70%.42.70%, and 31.11%) missed items 2, 7, 9.1, 9.2, & 16.2.4 as compared to the experimental group (11.90%, 23.81%, 21.43%, 16.67%, 2.38 %,) respectively.

The same interview questions were posted for participants from both groups "Have you missed or got it right items of theme two? Why or why not?" Their responses were stated as follows:

CG22: "*I missed most of the items for example items 2, 7,9.1, and 9.2 I made a mistake while converting the general form of the quadratic equation to vertex form to answer domain, rang, maximum height, time taken to reach maximum height, and x-coordinate of the vertex. In Item 15 I correctly answered the upward and downward effect of 'a' but interchangeably answered other effects of $a-$ for $+a$ bulged parabola for '$-a$' narrow parabola*". March, 27/2023.

CG38: "*When I attempt 16.2.2 I correctly sketch $f(x) = -x^2$ but missed all the sub-questions of $g(x) = -x^2 - 2x + 3$ except the domain because I did not correctly convert to vertex form*". March, 27/2023.

EG19: "*I answered most of the items but in item 16.2.2 the misplaced negative sign to convert vertex form due to this I misplaced the parabola horizontally. I usually left without doing word problem type questions in linear function but now I easily answered items 9.1 and 9.2 because the software helped me to connect the idea graphically while we were doing similar exercises*". March, 28/2023.

CG44: "*Sorry, teacher; I missed all the items except the items that request the domain and upward and downward opening effect of 'a' of a parabola*". March, 27/2023.

EG27: "*I answered all the questions because GeoGebra software helped me to visualize both effects of 'a' easily by moving slider 'a 'how the parabola opens dawn or opens up, how to compute the maximum height of a parabolic type graph and flight time. Furthermore, how the axis of symmetry of a parabola passes through vertex in animation form and different colors*". March, 28/2023.

EG33: "*Thank you, teacher, I am satisfied with my score I have answered all the questions. The software helped me to memorize easily what we had learned in class because it provided an opportunity to correct my work and to visualize the effects of parameters a, b, and c while sketching the parabola by moving*

the slider and easily comprehending how maximum value of a parabola linked to real life situation". March, 28/2023.

From the above interviewees' data, one can see misconceptions of students in the control group in two areas, the first one is the effect of 'a' in vertex form of the equation of a parabola *i.e.* $a(x+b)^2+c$ some students in the control group think that if a is positive the parabola open outward and if negative narrowed. The second misconception was in converting the general form equation of parabola to vertex form equation but this was not observed in the experimental group because only one student missed this item. This variation is nothing but attributed to the intervention using the GeoGebra applet.

To check the test of significance independent T-test was computed. Among nine items seven items namely 2, 7, 9.1, 9.2 15, 16.2.2, and 16.2.4 are statistical for p = 0.0055 < 0.05. Whereas the remaining two items were not statistical since p = 0.5685 > 0.05.

Students' Overall Score in Theme 3: Interpret and Use the Graph of a Quadratic Function

Eleven questions measured students' performance related to this learning theme and compared their scores in both groups. The items were number 3, 4, 7, 10.1, 10.2, 10.3, 10.4, 11.1, 11.2, 11.3 and 11.4. Table 4 depicts students' score statistics.

As portrayed in the table a number of respondents (46.67 %) from control group missed items of outcome four. Whereas 89.86% of respondents from experimental group answered items of outcome four correctly indicating that experimental group were perform better than the control group.

Table 4: Mean score of students for items in theme three

Theme three Item numbers	Respondents Type							
	Control Group(N = 45)				Experimental Group(N = 42)			
	Missed the item		Answered the item correctly		Missed the item		Answered the item correctly	
	Count	%	Count	%	Count	%	Count	%
3	20	44.44	25	55.55	5	11.90	37	88.10
4	22	48.89	23	51.11	0	0.00	42	100.0
7	22	48.89	23	51.11	10	23.81	22	76.19
10.1	22	48.89	23	51.11	5	11.90	37	88.10
10.2	26	57.78	29	42.22	5	11.90	37	88.10
10.3	15	17.78	30	82.22	5	11.90	37	88.10
10.4	22	48.89	23	51.11	3	2.38	39	97.62
11.1	16	35.56	29	64.44	5	11.90	37	88.10
11.2	30	66.67	15	33.33	5	11.90	37	88.10
11.3	18	40.0	27	60	4	9.52	38	90.48
11.4	25	55.56	20	44.44	2	4.44	40	95.56
Aggregated mean		46.67		43.33		10.14		89.86

The same form of interview questions was raised "**Have you missed or got it right items of theme three? Why or why not?**" While interviewees CG22, CG38, and CG44 of the control group responded as "*Yes, missed majority*", "*Totally I missed items 10 and 11, I answered the matching items by giving a double answer for question 10 but for question 11, I answered interchangeably wrong answer and I knew my mistake later*", and "*I missed all the items except item four which requests the axis of symmetry*" respectively, EG33, EG27, and EG19 form the experimental group responded as "*Yah, I have had no trouble attempting the entire question*", "*Yes I answered all the questions*" and "*I answered most of the items except question seven missed it by committing a minor mistake while hurrying up*", respectively.

The following are sample of the scripts from the interviews:

CG22: "*Yes, missed majority. For* $y = 3(x-5)^2+4$ *I have to find the value of* x *which makes* $x-5=0$, *but since the value outside bracket is* $+4$ *I had taken* -5 *directly. For item 10 select two answers for a question because I know the fact that if a is positive the parabola opens up if negative opens down so G1 and G2 for 10A & 10C lately I understand the difference*". March, 27/2023.

EG27: "*Yes I answered all the questions because GeoGebra software helped me to visualize both effects of 'a' easily by moving slider 'a' 'how the parabola narrows or bulges out for the negative or positive value of 'c' the parabola moves vertically up or down because repeatedly exercised both in-class exercise and worksheet questions*". March, 28/2023.

One can see students' misconceptions in three areas. The first one was failing to identify the axis of symmetry in the given vertex form equation. For the equation $y = 3(x-5)^2+4$ many students from the control group unlike the experimental group responded as if x = –5 which is wrong. The other misconception noticed was the inability to identify the effect of 'a' in items 10.1 – 10.4 of a quadratic function. Many students from the control group responded by giving double answers for a single question missing the narrowing or widening effect of 'a' as seen in Figure 6. The next misconception that happened was the inability to identify the role of the variable 'c' clearly in the equation ax^2+c type in items 11.1 – 11.4. As seen in Figure 6, if 'c' is positive, they answered the graph shift vertically opposite to 'c' and vice versa which is true for the effect of 'b' in a horizontal shift.

To check the test of significance t-test was computed and revealed that there were statistically significant differences between the two groups at an alpha level of 0.05 (N= 45, M = 0.51818; N = 42, M = 0.8927; T (85) = -6.05, p = 0.004 < 0.05). The average mean score of the experimental group was 0.8927, which was greater than the control group (M= 0.51818), indicating that the ones who learned using Geogebra outperformed better than their counterparts. In other words, the misconceptions observed in the control group were not problematic in the experimental group, indicating that the intervention greatly reduced misconceptions.

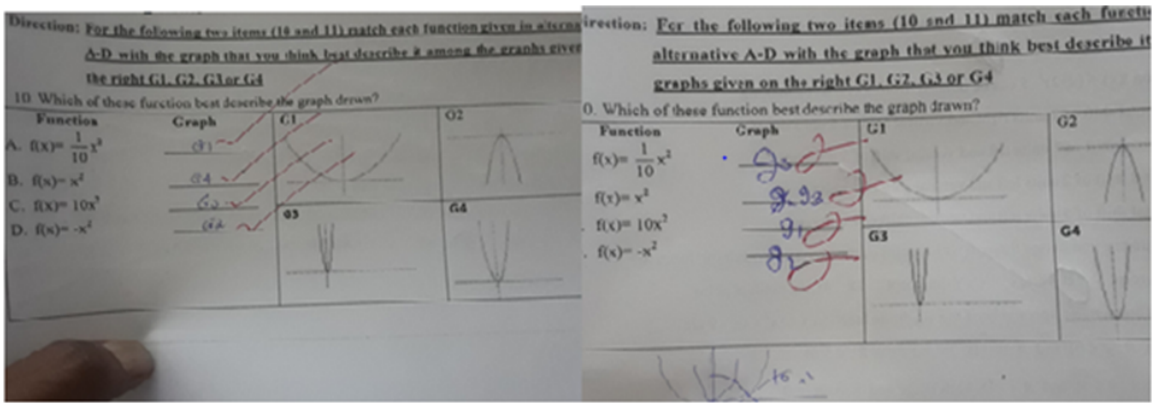

Figure 6: Scanned image of the works of students for items in theme 3

Students Score in Theme 4: Applying the Shifting Rules to Graph Quadratic Functions

In item 16.1.2 of theme four, students were asked to sketch the graph of $g(x) = (x-3)^2+2$ from parent function $f(x) = x^2$ applying a shifting rule. The cross-tabulation is illustrated in Table 5.

As portrayed in Table 5, 35.87% of the control group missed items of theme four. Whereas 81.97 % of respondents from the experimental group answered items of theme four correctly indicating that the experimental group performed better than the control group. Specifically for items 16.1.2, 16.3, and 16.2.2, since they have a different scoring key, they have been analyzed separately. Hence, for item 16.1.2 majority of students, 28(66.67%)

from the experimental group sketched the graph $g(x) = (x-3)^2 + 2$ without missing both vertical and horizontal shifts correctly whereas 24(53.33%) of the control group respondents failed to sketch appropriately, only 13(28.89%) of them tried sketching one shift correctly. Only 8(17.78%) of them attempted correctly.

Table 5: Mean score of students for items in theme four

Theme–items	Respondents Type	N	Missed		Attempt partly		Attempt at large		Fully Answered	
			N	%	N	%	N	%	N	%
5	Control Group	45	14	31.11	-	-	-	-	31	68.89
	Experimental Group	42	6	14.29	-	-	-	-	36	85.71
8	Control Group	45	22	48.89	-	-	-	-	23	51.11
	Experimental Group	42	5	11.90	-	-	-	-	37	88.10
16.1.1	Control Group	45	8	17.78					37	82.22
	Experimental Group	42	6	14.29					36	85.71
16.1.2	Control Group	45	24	53.33	13	28.89	-	-	8	17.78
	Experimental Group	42	2	4.76	12	28.57	-	-	28	66.67
16.1.3	Control Group	45	25	55.56	11	24.44			9	20.0
	Experimental Group	42	2	4.76	12	28.89			28	66.67
16.2.1	Control Group	45	6	13.33	-	-	-		- 39	86.67
	Experimental Group	42	2	4.76	-	-	-		- 40	95.24
16.2.2	Control Group	45	14	31.11	19	42.22	5	11.11	7	15.56
	Experimental Group	42	1	2.38	1	2.38	4	9.52	36	85.71
Aggregate mean	Control Group			35.87						48.89
	Experimental Group			8.16						81.97

Similarly, in item 16.1.3 students were asked to sketch the graph of $h(x) = (x+3)^2 - 2$ from parent function $f(x) = x^2$ applying the shifting rule and hence 28(66.67%) participants from the experimental group sketched the graph without missing both vertical and horizontal shifts correctly whereas 25(55.56%) of control group respondents failed to sketch appropriately, only 11(24.40%) of them tried sketching one side shift correctly and only 9(20.0%) of them attempted correctly.

In the same way, it was asked to sketch the graph of a quadratic function in general form rather than vertex form in item 16.2.2 to be sure students were not answering some related preceding objective items by chance. The question was asked to sketch the $f(x) = -x^2 - 2x + 3$ graph from parent function $f(x) = -x^2$ using the shifting rule. Thus, the majority of students (36 or 85.71%) from the experimental group sketched the graph without missing both vertical and horizontal shifts correctly, whereas only 7(15.56%) of the control group respondents sketched the graph appropriately. Some of them were missed, and 42.22% correctly converted general form into vertex form equation of quadratic function but failed to appropriately shift the graph. Concerning this, some scanned images of the works of students from exam papers were taken and illustrated as in Figure 7.

The images show students' misconceptions of the control group in two areas of quadratic function. The first one was overgeneralizing during shifting the graph $g(x) = (x-3)^2 + 2$. Some of them shift horizontally three units to a positive direction (opposite to -3) and incorrectly shift vertically two units to a negative direction (opposite to +2). The same procedure was followed for $h(x) = (x+3)^2 - 2$. Some others were shifted horizontally three units to the negative direction and two units vertically to the positive direction for $g(x) = (x-3)^2 + 2$ taking directly "-3" and "$+2$" So does for $h(x) = (x+3)^2 - 2$. The second misconception occurred in item 16.2.2

while converting $g(x) = -x^2 - 2x + 3$ into vertex form to identify the shifting direction. Some of them wrote $g(x)$ as $-(x^2 - 2x + 1 - 1) + 3 = -(x - 2x + 1) - 1 + 3 = -(x - 1)^2 + 2$ and some others wrote it as $-(x^2 + 2x + 1 - 1) + 3 = -(x + 2x + 1) - 1 + 3 = -(x + 1)^2 + 2$ but both of them were wrong. However, this was not a problem for the experimental group.

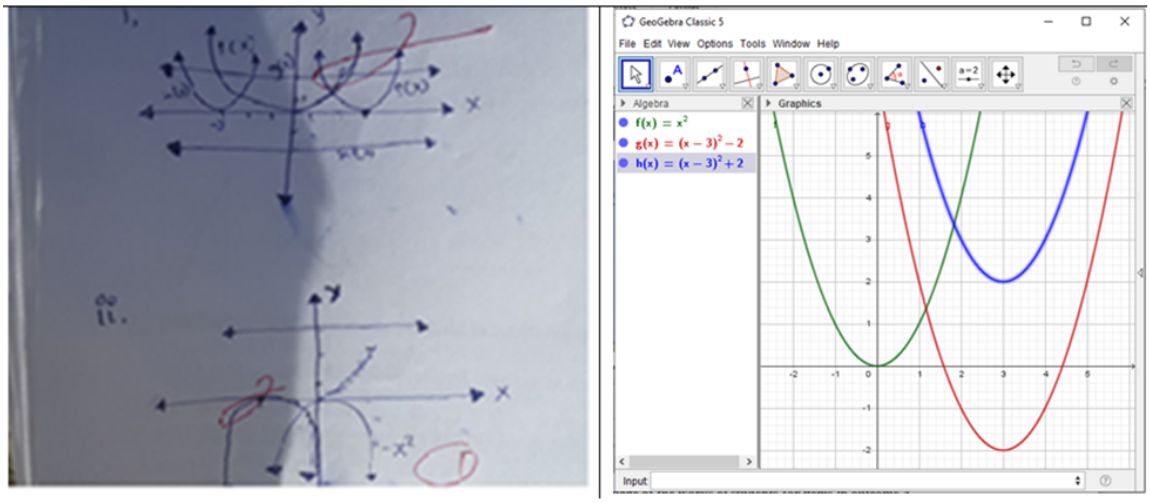

Figure 7: Scanned image of the works of students for items in theme 4

The researcher was eager to know the interviewees' reaction to items in theme four and posed the question "Have you missed or got it right items in theme four? Why or why not?" While the interviewees CG22, CG38, and CG44 from the control group responded as "*I missed almost the majority*", "*Missed items 16i and 16ii except for the domain and graphing* $f(x) = x^2$ and $f(x) = x^2$", and "*I missed some and replied some others correctly*" respectively, EG19, EG27, and EG33 from the experimental group responded "*I answered most of the items*", "*Yes I answered all the questions*", and "*Yah, I have had no trouble attempting the entire question*" respectively.

The following were sample of the scripts from the interviewees'

CG22: "*Sorry teacher I missed almost the majority especially failed to sketch the graphs correctly. For items of 16i mistakenly misplaced the graph of* $g(x) = (x - 3)^2 + 2$ *and* $h(x) = (x - 3)^2 - 2$ *in both direction. I interchanged horizontal shift for vertical shift and vertical shift for horizontal shift only I answered their domain. In the second part of item 16, I made a mistake while converting* $g(x) = -x^2 - 2x + 3$ *to vertex form*". March, 27/2023.

EG27: "*Yes I answered all the questions because GeoGebra software gives an opportunity to identify my mistakes during exercising in class and out of class in e- learning lab. By moving the parameters it enables me to understand the shape and behavior of the graph so I easily completed the exam even before elapse of time*". March, 28/2023.

Concerning the test of significance among seven items 4 items namely 8, 16.1.2, 16.1.3, and 16.2.2 were statistical (T(85) = -6.5605; p = 0.000 < 0.05). On the other hand, in the remaining items (5, 16.1.1, & 16.2.1) there were no statistically significant differences between the two groups (T (85) = - 1.233; p = 0.491 > 0.05). It seems that statistically there was no difference in sketching $f(x) = \pm x^2$ between the two groups.

3.3 Student's perceptions towards GeoGebra Software

The student's perception of GeoGebra software was determined using a questionnaire based on a five-point Likert scale. Table 6 summarizes students' responses to the questionnaire items.

From the data in Table 6, 283.4% agreed GeoGebra software creates an interesting environment in the classroom and 83.6 % mentioned they liked GeoGebra software to use in learning the quadratic functions. About 78.6 % of students believed GeoGebra software helps to reduce misconceptions while learning quadratic functions.

Table 6: Students' perception towards GeoGebra in teaching quadratic function

Attitude attribute items	Responses (%)				
	SA*	A*	Und.*	DA*	SD*
GeoGebra software creates an interesting environment in the classroom	7.1	76.3	7.1	9.5	0
I like GeoGebra software to use in learning quadratic function concepts	6.4	77.2	10.9	4.1	3.8
GeoGebra software helps to reduce misconceptions while learning quadratic function	2.4	76.2	4.7	11.9	4.8
GeoGebra software helps to increase mathematics achievement	14.8	73.8	4.8	6.6	0
GeoGebra software helps students to improve quadratic function knowledge	4.8	81.8	6.9	5.1	1.4
GeoGebra software helps to visualizes quadratic function with its graph	4.8	76.2	9.5	7.1	2.4
Mathematics classroom becomes more interesting if a teacher uses GeoGebra	7.1	73.8	2.4	16.7	0
GeoGera based learning helps students to remember for a long time about quadratic function & its graphs than the traditional method	4.8	79.2	8.9	4.8	2.4
Aggregated percentage mean	6.525	76.812	6.9	8.225	1.85
	83.337			10.075	

*SA = strongly agree, A= agree, Und = undecided, DA= Disagree and SD = strongly disagree.

The table also shows that about 88.6 % mentioned GeoGebra software helps to increase mathematics achievement and 86.6 % of students believed it helps students to improve their knowledge in quadratic function. Furthermore, 81% of them agree it enables them to visualize quadratic functions with its graph. Finally, 80.9% and 84 % found GeoGebra software more interesting if the teacher used GeoGebra and enabled them to remember quadratic functions and their graphs for a long time.

4 Discussions

According to the results, GeoGebra allowed the students to experience many scenarios of looking at quadratic functions, which raised the likelihood that they would grasp the intended learning objectives and hence increase the success rate and interest to learn. Due to the time-saving and interactive features of GeoGebra (Misini & Kabashi, 2021), students were able to graph more appropriate quadratic functions. It was clear that the time and effort students had saved allowed them to experience more situations and activities, which enhanced their learning. Another student-related feature revealed that students had a high level of collaboration since they frequently assisted one another. Working in groups and the level of interaction between students are two major advantages of using technology. Students were eager to show off their abilities and share what they had learned recently with their peers. Sometimes, students may be better teachers than teachers because they can interact with one another and actively help to clarify unclear or vague concepts. With this regard, the study (Praveen & Leong, 2013) aimed to examine the impact of GeoGebra on students' understanding of concepts in geometry and concluded that the software not only raised student test scores but also energized the classroom environment and emphasized the importance of cooperation and collaboration among students. The post-test result that shows the experimental group performs better than the control group in terms of achievement scores agrees with the results of studies in different contexts (for instance, Septian *et al*. (2020) in Indonesia and Övez (2018) in Turkey).

Thus, GeoGebra software is important in reducing students' misconceptions while teaching quadratic functions. In line with this, a study conducted by Gningue *et al*. (2014) compared the effects of teaching the concepts of pre-algebra and algebra using virtual manipulation and traditional methods and concluded that virtual manipulation can help students overcome misconceptions about algebra

and pre-algebra concepts. Another study by Ojose (2015) examined whether the use of the GeoGebra application allows students to determine and solve misconceptions in calculus classes and suggested that students who were taught using GeoGebra can draw function graphs better than those who did not. Besides, the study in calculus conducted in Ethiopia by Baye *et al*. (2021) concluded that the use of the GeoGebra applet enhanced students' visualization and improved their conceptual understanding of the limit. The same result is also observed in Bekene (2020), Saha *et al*. (2010), and Takači *et al*. (2015). Most of these studies are at higher education and the current study confirmed that a similar result is observed at the secondary school level.

Overall, the results of the post-test and data from the students' perception questionnaire revealed that GeoGebra-based teaching is more useful in teaching mathematics in secondary schools. During the lesson delivery, it was observed that students were more active and participated regularly in the experimental group. According to the students' view, GeoGebra was a new area of study in mathematics teaching at secondary schools. It visualizes quadratic function graphs easily with its algebraic form. Students in the experimental group overtook their peers in the control group.

This research also proved that students had positive attitudes toward the GeoGebra integrated instruction. In line with this Celen (2020) research on the topic "student opinions on the use of Geogebra software in mathematics teaching" found that students have a positive perception of using GeoGebra as it makes learning "fun and enjoyable". Similarly, Tamam and Dasari (2021) investigated "The use of Geogebra software in teaching mathematics". The purpose of the study was to synthesize the impact of using GeoGebra as a medium of teaching mathematics, and the study found that student's attitudes toward learning mathematics through technology improved, as well. Kim and Md-Ali (2017) also found that the GeoGebra integrated instruction of teaching shape and space concepts positively influenced students' engagement and desire to learn.

5 Conclusion

GeoGebra is an effective tool for teaching quadratic functions and enables to visualization effects of changing parameters and helps students to make connections with visual representation in learning quadratic functions and encourages them to solve mathematical problems related to course content at secondary schools. The results in this study have some implications for mathematics teaching and learning. Using technology such as GeoGebra changes the roles of both teachers and students in the teaching and learning process. When students use GeoGebra to learn quadratic functions, they assume active roles of receiving information from the teacher or textbooks. They actively make independent choices about how to move forward and are in a position to define their own goals, make their own decisions, and evaluate their own progress. Equivalently, GeoGebra could represent mathematics in ways that help students to understand concepts. When combined, these characteristics would allow teachers to enhance both what and how students learn. According to Bransford *et al*. (2000), when technology makes abstract ideas tangible, teachers can more easily build upon students' prior knowledge and skills, emphasizing connections among mathematical concepts, connecting abstractions to real-world settings, addressing common misunderstandings, and introducing more advanced ideas.

The findings of this study support the need for teachers to use blended teaching and learning strategies, which combine the use of talk and chalk instruction with computer technology (such as GeoGebra). The study further suggests that training teachers to utilize practical applications in teaching mathematics is an essential task that precedes using this software. Besides its desktop applications, its tablet and smartphone applications for Android, iPad, and Windows is an opportunity both for teachers and students to apply or practice based on their own style. This study and other studies in this area have shown the positive influence of mediating artifacts like GeoGebra to enhance the performance and attitude of students.

Limitation of the study

Due to financial constraints, this study has faced many limitations including the scope of the content, sample size, and duration of the intervention. Thus, future studies on the effect of integration of GeoGebra to reduce students' misconceptions and assess their attitude toward GeoGebra would demand comprehensive studies for longer periods, using far larger randomized sample sizes, at different schools of different composition and socio-economic status, which reflect the entire zone, region, or country level. This study further recommends in-depth research to investigate the root causes of misconceptions described in this study. In addition, future research should extend this study to other mathematics topics and grades to see if similar results are obtainable. Such studies' findings might help improve the quality of mathematics teaching and learning in Ethiopia.

Statements and Declarations

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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